***Fourier Transform***

* *Define Fourier Transform of f(x).*

If a function *f(x)* is defined on (-, it is continuous and piece-wise smooth, *f(t)* when |*t*| and *f(x)* is absolutely integrable, then Fourier Transform of *f(x)* denoted by F(α) is define by-

[*f(x)*] = F (α)

= eiαt dt

The inverse of F(α) denoted by [F(α)] is given by-

[F(α)] = f(x) = e-iαx dα

Note: Fourier Transform is also given as :-

[*f(x)*] = F (α) = eiαt dt

And the inverse of F(α) is given by-

f(x) = e-iαx dα

or, [f(x)] = F (α) = e-iαt dt

and f(x) = eiαx dα

**Problem 1**: *Find the Fourier Transform of f(x) = e-|x|*

Solution:

[*f(x)*] = F (α) =

= + }

[|t| = t, when t≥ 0 and |t| = -t , when t<0]

= + }

=

=

=

=

Inverse of F.T is –

*f(x)* = e-iαx dα

= dα

**Problem 2 :***Find the Fourier Transform of*

*f(x) =*

Solution:

[*f(x)*] = eiαt dt

=

=

=

=

=

=

=

=

=

*Properties of Fourier Transform*

(1) Fourier Transform is linear :

i.e. [ *af(x)* + *bg(x)* ] = *a*  [ *f(x)* ] + *b* [ *g(x)* ]

= *a*F(α) + *b*G(α)

**Proof**:-

[ *af(x)* + *bg(x)* ] = { +}

= *a* {} + *b* {}

= *a* [ f(x) ] + *b* [ g(x) ]

= *a*F(α) + *b*G(α)

**(2) Shifting property:-**

[ *f (t-c)* ] =  [ *f(t)* ]

**Proof:-**

[ *f(t-c)* ] = 

Let *t-c=u*  *t=u+c*

 *dt = du*

= 

= 

= {}

=  F(α)

(3) Scaling property:-

[ *f(ct)* ] =  ()

Proof:-

[ *f(ct)* ] = 

Let *ct = u*

 *dt* = 

=  

=  

=  ()

(4) Differentiation:- If *f (t)* is piece wise smooth on (-,) and

*f (t)*  0 as |*t*| , and *f (t)* and are absolutely integrable, then

[] = (*-iα*) [ *f(t)* ]

Proof:-

[] = 

=  { [ *f(t)*  - iα}

= 0 + (-iα) 

= (-iα ) [ *f(t)* ]

Similarly ,

[] = (-iα) []

=(-iα) (-iα) []

= (-iα ) [ *f(t)* ]

Proceeding in this way –

[] = (-iα) [ *f (t)* ] ; where, n = 1,2,3,………………..

**●** Find the Fourier transform of 

Solution:-

(α) = 

= 

 = 

=  ¯¯¯¯ Let (x-iα) = y

* *dx*= *dy*

 =

=  (Ans).



 \* => y=t

 => 

 =

 = г()

 =

 = 

* **Find the F.T of**

*f(x)* =

Hence evaluate

() dα

() d

soln

By the definition of Fourier transform, we have

[f(x)] =

= + +

= 0 + + 0

=

= ()

=

Taking the corresponding inversion formula –

*f(x)* = d

=

*f(x)* = () d

2 = () d

= () d

Equating real parts from both sides, we get

=

=

Again putting *a*=1 and *x*=0, we get

d

d =

* **Find the Fourier transform of**

And (i) evaluate the integral

(ii) From Perseval’s identity, Evaluate

**Solution:**

(i)

Ϝ [*f(x*)] =

* + +
* 0 + + 0
* + 2
* 0 + 2

i.e. F(α) = 4

Taking the corresponding inversion formula-

*f (x)* =

* =
* =
* =

Now equating real parts from both sides, we get

=

* =

Putting , we get

= =

* =
* =

Changing variable we get

=

(Ans.)

(ii) From Perseval’s identity, we have

=

* =
* =
* 2 =
* =
* =
* =
* =

= (Ans.)

**Convolution**

If two function *f(x)* and *g(x)* are defined on (-∞, ∞), then the convolution of *f(x)* and *g(x*) is denoted by and is define by-

=

**Convolution Theorem of Fourier Transform:**

The Fourier Transform of the convolution of *f(x)* and *g(x)* is the product of the Fourier Transform of *f(x)* and the Fourier Transform of *g(x)*,

i.e.

**Proof:**

By the definition of Fourier Transform, we have

Then we get,

By changing the order of the integral,

Let,

and

* **Find the Fourier sine transform of ; x>0**

**Solution:**

=

=I (say)

Then I =……………(1)

Differentiate w.r. to

=

=

=>

=> I = [integrating]

When ,this gives I=c

But I=0,for by(1) so that c=0

Therefore ,I=

i.e.

* **Find the Fourier cosine transform of**

**Solution:**

=

= I (say)

Then I= =…………………..(1)

Therefore,

=

=

= -

=- ………………………..(2)

Differentiating again,

=

=I

1. E. is:

So, m=

I=A+…………………………..(3)

= A-…………………………..(4)

When (1)=> I =

=

=

Therefore ,(3)=> I = A+B

So, A+B=…………….(5)

Also when

(2)=>

(4)=>

So, A-B=-………………………..(6)

From equations (5) & (6),

A=0

B=

From (3)

I= 0+

=>I=

=>

So,

**# Find the Fourier sine and cosine transform of e-ax, a>0**

**Soln:**

Fourier sine transform:

Fs [f(x)] = Fs () =

= sin dx

= [-a sin

=

Fourier cosine transform:

*Fc*[*f(x)*] = *Fc* () = cos dx

= cos dx

= [- cos

=

# Find the Fourier sine and cosine transform of the function

Soln:

For Fourier sine transform,

Fs [f(x)] = Fs () =

=

= [-

= (1-cosαa)

For Fourier cosine transform,

Fc () = cos dx

=

= [

=

**Finite Fourier Transform**

# Find the (a) finite Fourier sine transform and

(b) finite Fourier cosine transform of the function

Soln:

(a) Fs [f(x)] = Fs () =

=

= [- +

= cosαπ + [

= cosαπ + 0

= cosαπ

(b) Fc [f(x)] = Fc () =

=

= [

= 0 + [

= [ cosnπ - 1]